We introduce a recursive (“anytime”) distributed algorithm that iteratively restructures any initial spanning tree of a weighted graph towards a minimum spanning tree while guaranteeing at each successive step a spanning tree shared by all nodes that is of lower weight than the previous. Each recursive step is computed by a different active node at a computational cost at most quadratic in the total number of nodes and at a communications cost incurred by subsequent broadcast of the new edge set over the new spanning tree. We show that a polynomial cubic (linear) in the number of nodes bounds the worst (best) case number of such steps required to reach a minimum spanning tree and, hence, the number of broadcasts along the way. We demonstrate in simulation that this distributed, anytime nature of this algorithm is particularly suited to tracking minimum spanning trees in (sufficiently slowly changing) mobile ad hoc networks.

**Abstract**

**Problem Statement**

- Let $G = (V,E,W)$ be a weighted, undirected connected graph.
- Suppose that each node $v$ in $V$ is equipped with:
  - the (same, initial) spanning tree $S_0$ of $G$;
  - a list of its immediate neighbors and the associated edge weights in $G$.
- Then we seek a distributed recursive minimum spanning tree policy, initiated at an arbitrary, a priori designated “active” node $a_0$ in $V$, whose next, $k+1$ step, follows from step $k$ in $N$ with the following properties:
  - Local Optimization: at each step $k$, node $a_k$ computes a locally optimal restructuring of $S_k$ towards another spanning tree $S_{k+1}$ that has a lower edge weight sum;
  - Broadcast: at each step $k$, node $a_k$ broadcasts to all nodes the localized edge insertions and deletions that transform $S_k$ into $S_{k+1}$;
  - Token Circulation: at each step $k$, node $a_k$ activates only one adjacent node, $a_{k-1}$, in $V$ to guarantee mutual exclusion.

**Minimum Spanning Tree (MST)**

- A minimum spanning tree (MST) of $G = (V,E,W)$ is a connected acyclic subgraph of $G$ with the minimum sum of edge lengths.

**Recursive Distributed MST (RDMST) Algorithm**

**Complexities Analysis**

- **Lemma:** The locally optimal restructuring of any spanning tree $S$ of a connected graph $G = (V,E,W)$ at a node $u$ in $V$ can be computed at most in $O(\text{deg}(u))$ time.$^5$
- **Theorem:** The RDMST algorithm initiated from any spanning tree of a graph $G = (V,E,W)$ terminates with a MST of $G$ using at least $O(V)$ and at most $O(|V||E|)$ messages.
- **Summary:**
  - $O(|V|)$ Message Complexity of RDMST $\leq O(|V||E|)$
  - $O(|V|\log|V|)$ Message Complexity of DMST $\leq O(|V|\log|V|+|E|)$

**Numerical Evaluation**

- **Task:** Continuous Tracking of MST
- **Mobility Model:** Random Waypoint (Random Walk yields similar results.)
  - (Work Space: 100m x 100m, Speed Range: [0 10] (ms), Pause Duration = 5s)
- **Performance Measure:** Message Complexity (the total number of messages used to find an MST).

**Conclusion & Future Work**

- We introduce a simple anytime distributed MST algorithm for a fixed network with the intent to apply it to online tracking of MSTs in dynamic networks.
- The recursive nature of this algorithm lends itself to real time dynamic settings by permitting nodes to stop routing management at any time and resume arbitrarily later.
- A promising extension is the design of a collective decision rule combining the strengths of both constructive and recursive MST algorithms.
- In the longer term, we are planning to generalize this proposed algorithm to a broader class of adaptive hierarchical routing protocols.

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**References**