Introduction

In robotics, it is essential to model and understand the topologies of configuration spaces in order to design provably correct motion planners. The common practice in motion planning for modelling configuration spaces requires either a global, explicit representation of a configuration space in terms of standard geometric and topological models, or an asymptotically dense collection of sample configurations connected by simple paths. As a new alternative approach, we propose the use of clustering for closing the gap between these two complementary approaches to combine their strengths.

Clustering enables us to discover hidden intrinsic structures in generally complex shaped and high-dimensional configuration spaces. We argue that the intrinsic local structures in configuration spaces that are identified by clustering can be exploited to design computationally efficient, provably correct motion planners.

What Does Clustering Offer?

Traditionally an unsupervised learning method, clustering offers automated tools to discover coherent groups in configuration spaces to model their unknown global organizational structure (e.g., hierarchical clustering), and to determine collision-free local neighborhoods of robot configurations (e.g., partitional clustering) [1].

On a more conceptual level, clustering can be viewed as a symbolic abstraction relating the continuous space of configurations to the (combinatorial) space of clustering models (e.g., cluster hierarchies and finite set partitions).

In consequence, explicit relations between clustering models can be exploited to reduce the complexity of high-level motion planning.

Another characteristic use of clustering is for locality identification. One can utilize clustering to identify a collision-free neighborhood of a robot that also captures the local geometric structure of the configuration space around the robot's instantaneous position.

Hierarchical clustering offers a natural abstraction for ensemble task encoding and control of multirobot systems in terms of precisely yet flexible organizational specifications at selectively multiple resolutions. This abstraction intrinsically suggests a two-level navigation strategy for coordinated motion design [6]:

1. At the low-level, perform finer adjustments on configurations by using hierarchy preserving vector fields [3];
2. At the high-level, resolve structural conflicts between configurations by using a discrete transition policy in tree space [5].

The connection between these two levels is established by an optimal selection of a portal configuration supporting two adjacent hierarchies [4].

Hierarchical Abstraction

A relation from the continuous configuration space to the abstract space of binary hierarchies

Theorem: The homotopy model of configurations sharing the same cluster hierarchy is a generalized torus, \( (S^d)^2 \).

Computational Properties

- Hierarchical preserving navigation is computable in \( O(n^2) \) time.
- Navigation in tree space requires at most \( O(n^2) \) steps, each step costing \( O(n) \) computations.
- Computationally, the complexity of the motion planning algorithm is at most \( n^2 \).
- Each robot's motion planning in tree space is \( o(n) \) per step.

Encoding Collisions via Robot-Centric Voronoi Diagrams

We introduce a new, robot-centric application of Voronoi diagrams to encode robot collisions exactly by exploiting the local structure of configuration spaces around a robot configuration [7,8]. This also enables us to determine a safe convex neighborhood of a robot configuration.

We show that the continuous feedback motion toward the metric projection of a desired goal onto the robot's convex cell steers almost all robot configurations to the goal in environments cluttered with spherical obstacles, while avoiding collisions along the way [7]. We observe that the robot balances its distance to all proximal obstacles while navigating toward the goal.

In distributed mobile sensing applications, Voronoi diagrams are often utilized for solving sensory task assignment and for modelling group heterogeneity in actuation, sensing, computation and energy sources [2]. In addition to these usages, we tailor Voronoi diagrams to encode collisions in a heterogeneous group of disk-shapes robots.

Based on standard coverage control of point robots [2], we propose a constrained coverage control law for heterogeneous disk-shaped robots that solves the combined sensory coverage and collision avoidance problem [8]. We further introduce a congestion management heuristic for unassigned robots to hasten the assigned robots’ progress.

Conclusion

We introduce the use of clustering for modelling configuration spaces and for design of provably correct motion planners. This new philosophy for modelling configuration spaces, still in its infancy, promises results for closing the gap between standard configuration space and sampling-based motion planning approaches. We demonstrate some potential applications of clustering to the problem of feedback motion planning and control. We believe that these nontrivial applications of clustering to robot motion design only scratch the surface of its long-term potential.

References

Acknowledgement: This work was funded in part by the Air Force Office of Science Research under the MURI FA9550-10-1-0567.