A Recursive, Distributed Minimum Spanning Tree Algorithm for Mobile Ad Hoc Networks

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Abstract

We introduce a recursive ("anytime") distributed algorithm that iteratively restructures any initial spanning tree of a weighted graph towards a minimum spanning tree while guaranteeing at each successive step a spanning tree shared by all nodes that is of lower weight than the previous. Each recursive step is computed by a different active node at a computational cost at most quadratic in the total number of nodes and at a communications cost incurred by subsequent broadcast of the new edge set over the new spanning tree. We show that a polynomial cubic (linear) in the number of nodes bounds the worst (best) case number of such steps required to reach a minimum spanning tree and, hence, the number of broadcasts along the way. We demonstrate in simulation that the distributed, anytime nature of this algorithm is particularly suited to tracking minimum spanning trees in (sufficiently slowly changing) mobile ad hoc networks.

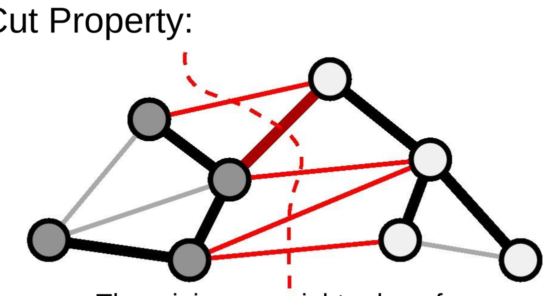
Problem Statement

- Let G = (V,E,W) be a weighted, undirected connected graph.
- Suppose that each node v in V is equipped with:
  - the (same, initial) spanning tree S_0 of G;
  - a list of its immediate neighbors and the associated edge weights in G.
- Then we seek a distributed recursive minimum spanning tree policy, initiated at an arbitrary, a priori designated "active" node a_0 in V , whose next, k+1 step, follows from step k in N with the following properties:
  - Local Optimization: at each step, k, node a_k computes a locally optimal restructuring of S_k towards another spanning tree S_{k+1} that has a lower edge weight sum;
  - Broadcast: at each step, k, agent a_k broadcasts to all nodes the local edge insertions and deletions that transform S_k into S_{k+1};
  - Token Circulation : at each step, k, agent a_k activates only one adjacent node, a_{k+1} in V to guarantee mutual exclusion.

Minimum Spanning Tree (MST)

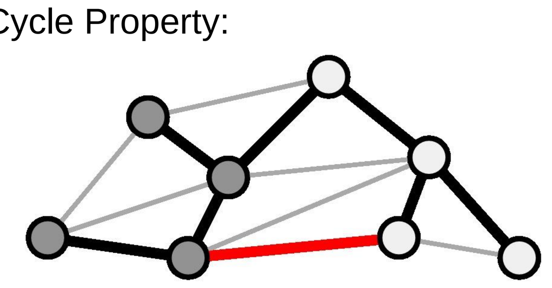
- A minimum spanning tree (MST) of G = (V,E,W) is a connected acyclic subgraph of G with the minimum sum of edge lengths.

Cut Property:



The minimum-weight edge of a cut is contained in some MST.

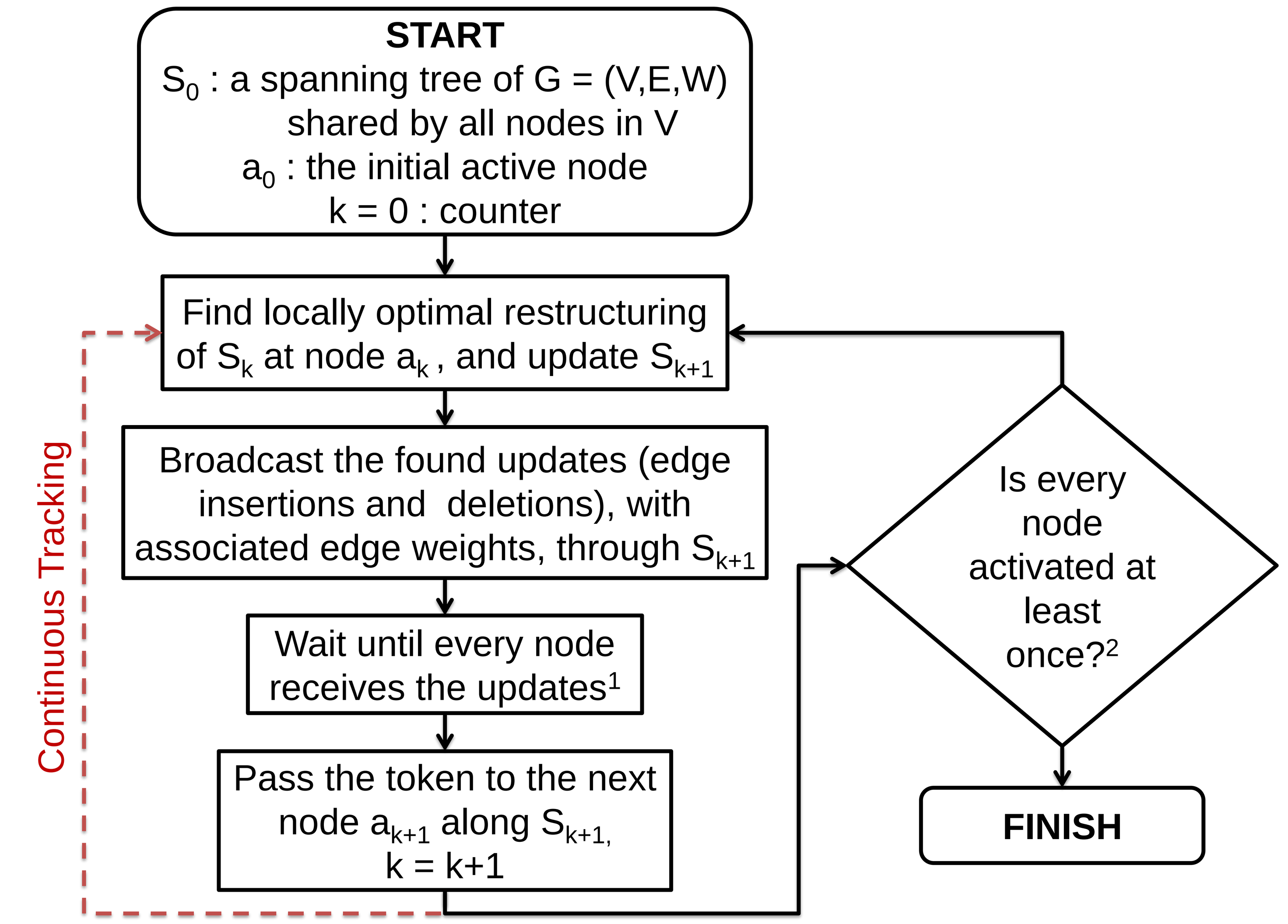
Cycle Property:



The maximum-weight edge of a cycle does not belong to any MST.

- *Definition:* A spanning tree S of G = (V,E,W) is *locally optimal* at a node v in V if S satisfies the cycle property locally at node v (based on its local connectivity in G).
- *Corollary:* A spanning tree S is a MST of G if and only if it is locally optimal at every node of G.
- Prior work: Distributed MST Algorithm (DMST) by Gallager et al [1]:
  - uses the cut property to collectively construct an MST of G,
  - requires at least O(|V|log|V|) and at most O(|V|log|V|+|E|) messages [1,2].

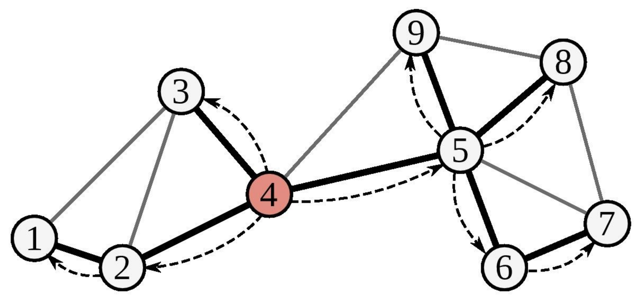
Recursive Distributed MST (RDMST) Algorithm



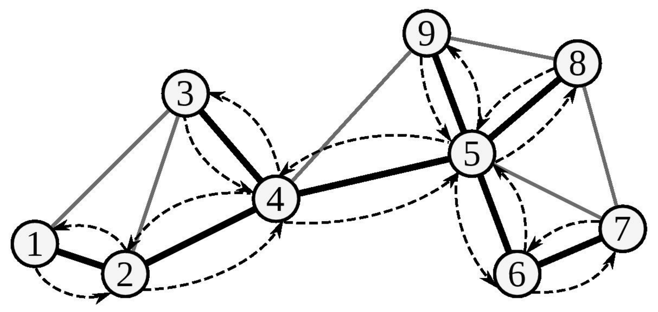
[1]This is required to guarantee the maintenance of a shared spanning tree and is achieved by passing a report message towards the active node along the spanning tree.

[2]Here an "activation counter" is used to make this decision. It is a register passed from node to node which is incremented by each node only upon its first activation.

- Broadcasting and Token Circulation:



Broadcasting (dashed) using a spanning tree (thickened) of a connected graph starting at node 4



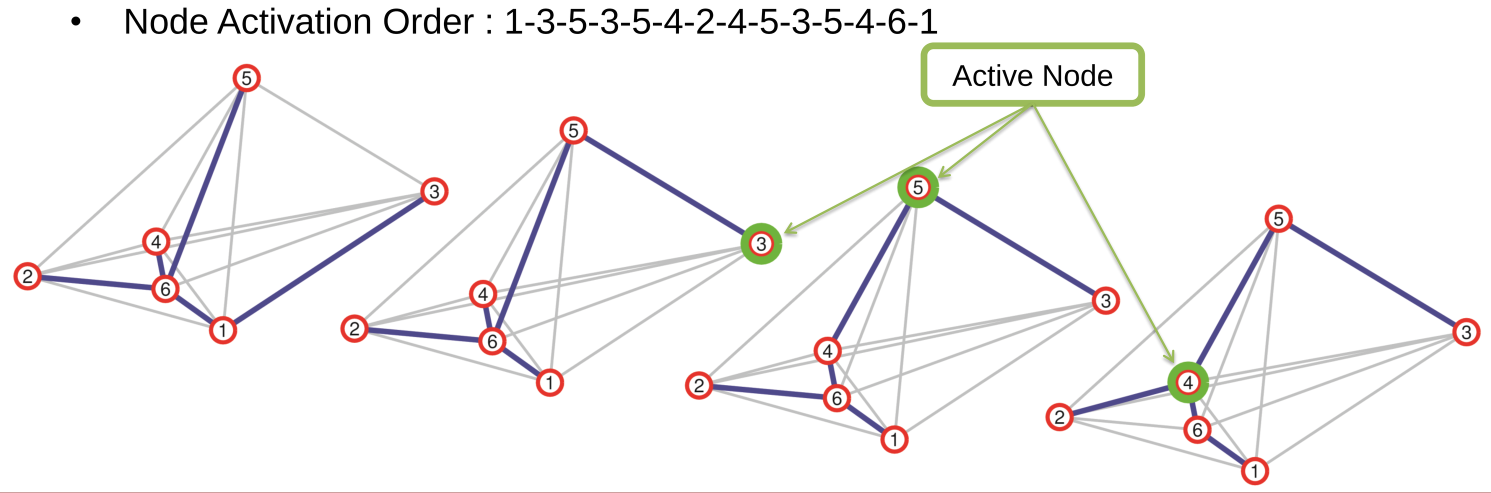
Token circulation using Euler Tour (dashed) of a spanning tree (thickened), which follows nodes 1-2-4-5-6-7-6-5-8-5-9-5-4-3-2 in circular order

- Complexities Analysis:
  - *Lemma:* The locally optimal restructuring of any spanning tree S of a connected graph G = (V,E,W) at a node u in V can be computed at most in O(|V|deg(u)) time[3].
  - *Theorem:* The RDMST algorithm initiated from any spanning tree of a graph G =(V,E,W) terminates with a MST of G using at least O(|V|) and at most O(|V| |E|) messages.
  - Summary:

$$O(|V|) \leq \text{Message Complexity of RDMST} \leq O(|V||E|)$$
$$\text{IA} \qquad\qquad\qquad\qquad \text{VI}$$
$$O(|V|\log|V|) \leq \text{Message Complexity of DMST} \leq O(|V|\log|V|+|E|)$$

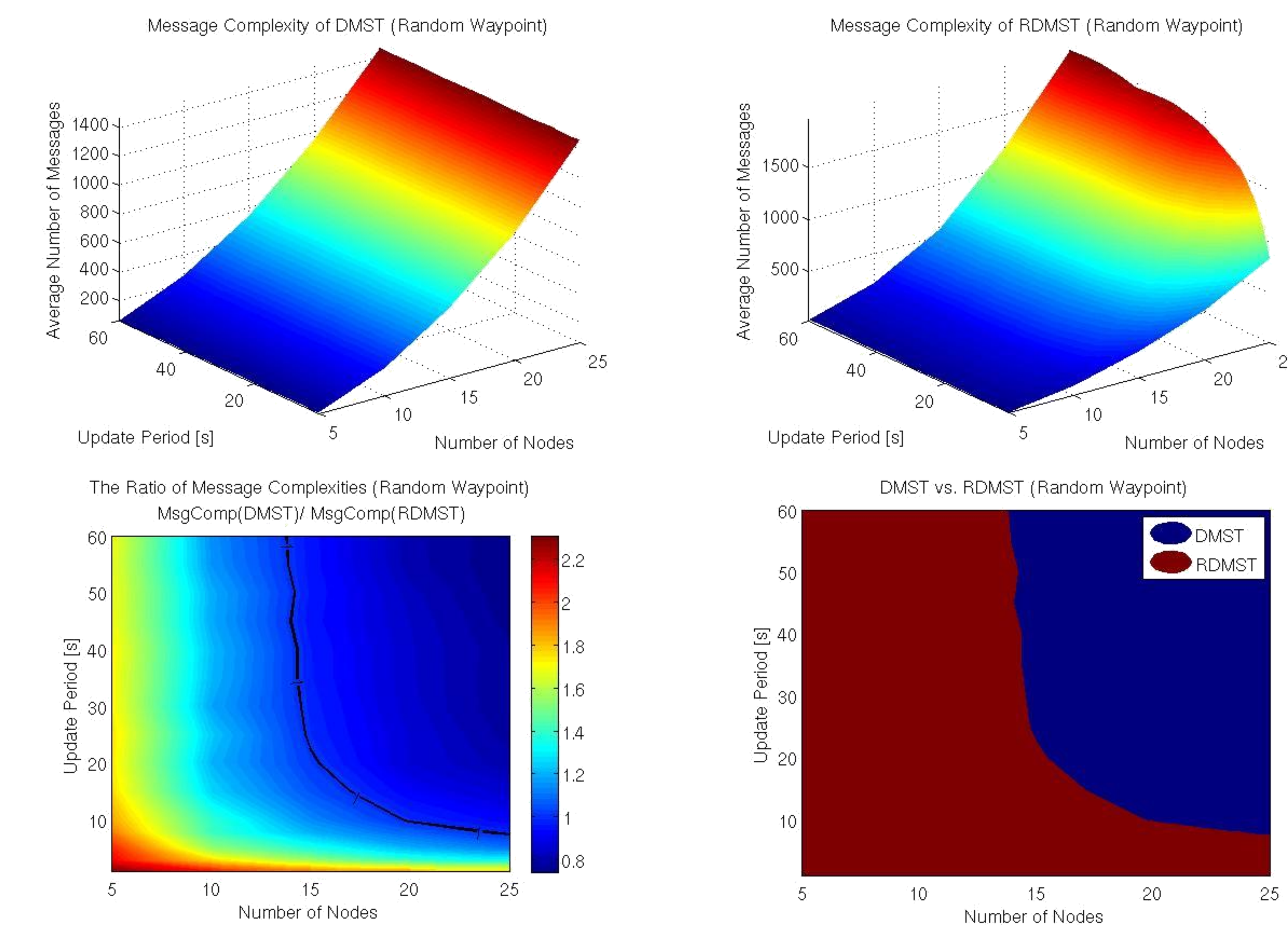
[3]deg(u) denotes the degree of node v in G.

- A Sample Sequence of Updates:
  - Node Activation Order : 1-3-5-3-5-4-2-4-5-3-5-4-6-1



Numerical Evaluation

- *Task :* Continuous Tracking of MST
- *Mobility Model :* Random Waypoint (Random Walk yields similar results.) *(Work Space : 100mx100m, Speed Range : [0 10] (m/s), Pause Duration = 5s)*
- *Performance Measure :* Message Complexity (the total number of messages used to find an MST).



Conclusion & Future Work

- We introduce a simple anytime distributed MST algorithm for a fixed network with the intent to apply it to online tracking of MSTs in dynamic networks.
- The recursive nature of this algorithm lends itself to real time dynamic settings by permitting nodes to stop routing management at any time and resume arbitrarily later.
- A promising extension is the design of a collective decision rule combining the strengths of both constructive and recursive MST algorithms.
- In the longer term, we are planning to generalize this proposed algorithm to a broader class of adaptive hierarchical routing protocols.

Acknowledgement

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References

[1] Gallager, R. G., Humblet, P. A., Spira, P. M. (1983). A distributed algorithm for minimum-weight spanning trees. ACM Transactions on Programming Languages and systems (TOPLAS).
[2] Korach, E., Moran, S., Zaks, S. (1984) Tight lower and upper bounds for some distributed algorithms for a complete network of processors. In Proceedings of the third annual ACM symposium on Principles of distributed computing (PODC).