Natural Control in Manufacturing
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Daniel E. Koditschek ¹

Center for Systems Science
Yale University, Department of Electrical Engineering

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Abstract

This paper reviews certain theoretical results in robot task planning and control obtained with the support of NSF Research Initiation Grant DMC-8505160. The "natural control" paradigm is reviewed and detailed attention is focused upon the specific task of robot navigation in a cluttered environment. The paper concludes with some comments concerning the problems which remain before this approach to robot command and control can be made practicable in a real manufacturing environment. A summary of formal results presented in the paper now follows.

The class of "navigation functions" simultaneously encodes the task of navigating amidst obstacles and automatically generates correct feedback control laws for this purpose as well. We show that the topology of navigation tasks precludes any stronger class of controllers (with respect to convergence properties). On the other hand we demonstrate that a member of this class must exist for any environment. Finally, we construct controllers for an increasingly realistic catalogue of environments by recourse to "deformation" of the true environment into a computationally simpler but topologically equivalent model.

1 Introduction

This paper offers a review of recent results in the theory of "natural control" for robotic systems, and concludes with some speculations concerning the implications for factory automation. The central observation is that since this control paradigm is intended to unify the description of abstract user goals with the imperatives of low level machine sensing and control, potentially large performance improvements with respect to predictability of outcome might attend the introduction of this methodology on the factory floor. Of course, there are a great many open questions to be addressed — both on the theoretical as well as the practical level — before this is possible.

A largely unspoken understanding seems to prevail in the field of robotics to the effect that methods of task planning must be unrelated to methods of control. The former belong to the realm of geometry and logic whereas the latter inhabit the the earthier domain of engineering analysis; geometry is usually associated with off-line computation whereas everyone knows that control must be accomplished in real-time; the one is a "high level" activity whereas the other is at a "low level". The result is a paradigm that calls for the generation of reference trajectories by some off-line computation: the system is then forced to track the reference signal by an on-line controller.

This article concerns a circle of ideas that, in contrast, intrinsically binds action and intention together in the description of the robot's task. A rather different control methodology replaces the role of feedforward reference signals in favor of task characterization via reference dynamics. In the present setting, the vector field representing the reference dynamical system is merely
"lifted" into the actual mechanical system. "Natural Motion", then, describes the resulting unforced response of the autonomous closed loop system. It will be seen that the capabilities of such purely error-driven systems may afford the command of a surprisingly sophisticated range of abstract goals, hence, deserve a much better hearing in the robotics and automation community than hitherto been accorded.

For present purposes, we shall restrict detailed attention to the specific task domain of robot navigation amidst obstacles. For the problem of moving in a cluttered space, being, perhaps, one of the most developed areas of robotics is also the domain within which the natural control methodology is most completely articulated. There is a growing body of work concerning robotic task domains involving static forces as well as geometry. In this context, fundamental work by Hogan [4] advances persuasive arguments for encoding general manipulation tasks in the form of "impedances", and these may be clearly related to the paradigm of natural control [9]. Moreover, even in robotic task domains involving dynamical environments, it begins to appear as though a geometric formalism will suffice to describe and control complex behavior such as throwing, catching, and juggling [1]. Nevertheless, we confine attention here to the purely geometric environments encountered in motion planning problems.

This paper is organized as follows. The next section provides a trivial example of a "natural controller" as the familiar PD algorithm of first year linear systems theory. The following section illustrates how the simple idea generalizes to complex motion planning problems by way of a review of some of our recent theoretical results. Finally, the paper concludes with some speculative comments concerning the possibilities for implementation of these ideas in an automated factory setting.

2 Example: An End-Point Task for a Simple Robot

Consider the "one-degree-of-freedom" robot — a single (revolute or prismatic) joint whose position, $q$, and velocity, $\dot{q}$, are measured exactly and instantaneously by a perfect sensor, actuated by a motor which delivers exact torque, $\tau$, according to the user's instantaneous command — and its dynamical equation, given by Newton's second law as

$$M \ddot{q} = \tau,$$  \hspace{1cm} (1)

where $M$ is the mass (or moment of inertia in the revolute case) of the robot link. Suppose the robot has been given the task of moving to a point, $q_d$, and remaining there. One might imagine splitting the task up into a "high level" geometric problem — find a curve in the jointspace, $c(t)$, which ends up at $q_d$ — and a "low level" control problem — find a control law, $\tau(t)$, which "forces" the robot to "track" the commanded behavior, $q(t) \rightarrow c(t)$. In much of the robotics literature, these two problems are solved independently. In the methodology under consideration, they are solved at the same time.
2.1 Geometry: Hill Climbing

The geometric aspect of this task may be represented by the following optimization problem. Conceive of a “cost” function on the jointspace, \( \varphi \), which assigns a scalar value to every position, vanishing uniquely at the desired “target” \( \varphi(q_d) = 0 \), and growing larger farther away. For example, the quadratic function, which we shall come to call a “Hook’s Law” cost,

\[
\varphi_{HL} \equiv \frac{1}{2} K_P [q - q_d]^2,
\]

would do nicely for all positive values of the scalar \( K_P \). If \( \varphi \) is continuously differentiable then it has a well defined negative gradient system,

\[
\dot{q} = -\text{grad} \, \varphi. \tag{2}
\]

Solutions of this differential equation follow the “fall line” of the “hill” defined by the cost function — i.e. at every position, the velocity of any solution curve is specified by the directional derivative of \( \varphi \). If \( q_d \) is a local minimum of \( \varphi \) it follows that all solutions of system (2) which originate in some neighborhood of that point, define curves which lead to that point. If, in addition, \( q_d \) is the only extremum of \( \varphi \), then every solution curve leads there. For example, in the case of the quadratic cost function, \( \varphi_{HL} \), the gradient system works out to be the scalar linear time invariant differential equation,

\[
\dot{q} = -K_P [q - q_d].
\]

Since \( q_d \) is a minimum, and the only extremum of \( \varphi \) to boot, it follows that this gradient system generates a solution to the geometric “find path” problem from any starting position.

2.2 Control: Energy Dissipation

Faced with the particular problem of navigating from some initial position, \( q_0 \), to the target, \( c_d \), one might now define a reference trajectory, \( c(t) \), by solving the gradient differential equation, (2), for the initial condition, \( q_0 \), and then attempt a tracking control. Instead, we will re-interpret the cost function, \( \varphi \), as a potential function, and introduce a control law which achieves the desired result with no recourse to explicit solutions of the original gradient system, (2).

If we are to interpret \( \varphi \) as a potential function, we must form the total energy by taking its sum with the kinetic energy,

\[
\eta \equiv \frac{1}{2} \dot{q}^T M \dot{q} + \varphi,
\]

and then apply the Lagrangian formalism,

\[
\left[ \frac{d}{dt} \frac{\partial}{\partial \dot{q}}, \frac{\partial}{\partial q} \right] \eta = f_{\text{ext}},
\]

(where \( f_{\text{ext}} \) denotes all non-conservative forces) to obtain the Newtonian law of motion,

\[
M \ddot{q} + \text{grad} \, \varphi = f_{\text{ext}}.
\]
If $f_{\text{ext}}$ represents the effect of a dissipative force, say a Rayleigh damper, $f_{\text{ext}} = -K_D \ddot{q}$, where $K_D$ is positive, then the total energy must decrease:

$$\dot{\eta} = -K_D \ddot{q}^2.$$

In consequence, it seems intuitively plausible, and will be made rigorously clear below, that $(q, \dot{q})$ converges to $(q_d, 0)$ from some neighborhood of that point in phase space — the space of positions and velocities — as long as $q_d$ is a minimum of $\varphi$.

The final equation of motion resulting from this formulation is

$$M \ddot{q} + K_D \dot{q} + \nabla \varphi = 0. \quad (3)$$

It is clear that (1) may be made to look like (3) by assignment of the control law,

$$\tau \triangleq -K_D \dot{q} - \nabla \varphi.$$

We have thus “lifted” the vector field on the configuration space (2) to the doubly dimensioned phase space. As claimed, if $q_d$ is a local minimum of $\varphi$, then, under the influence of this control law, our robot is guaranteed to approach $(q_d, 0)$ asymptotically from any initial state, $(q_0, \dot{q}_0)$, which is sufficiently close to $(q_d, 0)$. In the particular case of a Hook's Law spring potential, $\varphi_{HL}$, this control strategy corresponds exactly to the time honored “proportional-derivative” feedback control strategy of linear systems theory,

$$\tau \triangleq -K_D \dot{q} - K_P [q - q_d],$$

with the familiar closed loop dynamics,

$$M \ddot{q} + K_D \dot{q} + K_P [q - q_d] = 0.$$
3 Review of Technical Results

The idea of using "potential functions" for the specification of robot tasks was pioneered by Khatib [7] in the context of obstacle avoidance, and further advanced by fundamental work of Hogan [4] in the context of force control. The methodology was developed independently by Arimoto in Japan [11], and by Soviet investigators as well [12]. This methodology provides the exemplar of the natural control paradigm, and this section reviews some new formal results in the area. First, we consider the limit behavior of generalizations of (2), and the extent to which such behavior is copied by a mechanical system. Next we define the class of navigation functions — a class of potential functions with certain additional technical properties which ensure the correctness of the control laws vis a vis the problem of moving in a cluttered environment — and describe the progress we have made toward their construction.

3.1 Gradient and Lagrangian Limit Sets

In general, gradient dynamical systems have extremely simple limiting behavior.

**Proposition 3.1 ( [8] )** Let $\varphi$ be a continuously differentiable Morse function on the compact Riemannian manifold, $\mathcal{J}$. Suppose that $\text{grad } \varphi$ is transverse and directed away from the interior of $\mathcal{J}$ on any boundary of that set. Then the negative gradient flow has the following properties:

1. $\mathcal{J}$ is a positive invariant set;
2. the positive limit set of $\mathcal{J}$ consists of the critical points of $\varphi$;
3. there is a dense open set $\mathcal{J} \subset \mathcal{J}$ whose limit set consists of the local minima of $\varphi$,

A Lagrangian system is defined by applying Lagrange's operator (from Section 2) to the difference between the kinetic and potential energy of a mechanical system. Adding a damping term to the change in momentum terms results in a dissipative Lagrangian system. It has been known for at least a century that the local limiting behavior of good potential fields, $\varphi$ "lifts" to a dissipative Lagrangian system to copy that of the gradient system.

**Theorem 1 ( Lord Kelvin (1886) [16][ §345 ] )** If $q_0$ is a local minimum of $\varphi$ in $\mathcal{J}$, then $(q_0,0)$ is a stable equilibrium state of a dissipative mechanical system in $T\mathcal{J}$.

In point of fact, the lifted limit behavior may be established globally as well.
Theorem 2 ([8]) Let \( \varphi \) be a twice differentiable admissible Morse function on the compact set, \( J \). Let \( \eta \) denote total energy, and

\[
\eta_0 \triangleq \inf \varphi | \partial J
\]

The set of "bounded total energy" states

\[
\gamma^{\eta_0} \triangleq \{ v \in TJV : \eta \leq \eta_0 \},
\]

is a positive invariant set of a dissipative mechanical system within which all initial conditions excluding a nowhere dense set tend toward a point in the zero section of \( TV \) identified with a local minimum of \( \varphi \).

3.2 Navigation Functions

Let \( F \subset \mathbb{E}^n \) be a compact connected analytic manifold with boundary. A map \( \varphi : F \rightarrow [0, 1] \), is a navigation function if it is:
1. Analytic on \( F \);
2. Polar on \( F \), with minimum at \( q_0 \in F \);
3. Morse on \( F \);
4. Admissible on \( F \).

The formal justification for this definition is provided in [10]. For the present purposes, it suffices to point out that such functions have one and only one minimum, hence, according to Proposition 3.1 the gradient flow of such a function leads to that one minimum with probability unity [8, 10]. The construction of smooth vector fields with stronger convergence properties is topologically impossible [10]. Note that the insistence upon analytic (rather than merely smooth — i.e. infinitely differentiable) functions represents an attempt to preclude the necessity of writing computer programs. Analytic constructions will be much harder to obtain, in general, but since their local properties induce global properties — e.g. they cannot be "patched together" via partitions of unity as can merely \( C^\infty \) functions — the task specification and control algorithms which result will obtain from automatic parsing of closed form mathematical expressions (assuming, of course, we have avoided infinite sums and products as is seen to be the case below).

In consequence of Theorem 2, a mechanical system with this potential energy will approach the desired destination with the same probability. Thus, attention now turns to the possibilities for actually building such structures. Since, as has been mentioned above, topological obstructions preclude scalar functions with a single critical point, it seems reasonable to ask whether the slightly weaker requirement of a single minimum point might be prohibited as well. This worry gets put to rest immediately by the following result.

Theorem 3 ([10]) For every smooth compact connected manifold with boundary, \( M \), and any point, \( x_0 \in M \), there exists a \( C^\infty \) navigation function.
We now proceed with a review of the constructive results obtained to date. A sphere world is a compact connected subset of $\mathbb{E}^n$ whose boundary is formed from the disjoint union of a finite number, say $M + 1$, of $(n - 1)$-spheres. It follows that there is one large sphere which bounds the workspace,

$$\mathcal{W} \triangleq \{ q \in \mathbb{E}^n : \| q \|^2 \leq \rho_0^2 \},$$

and $M$ smaller spheres which bound the obstacles,

$$\mathcal{O}_j \triangleq \{ q \in \mathbb{E}^n : \| q - q_j \|^2 < \rho_j^2 \} \quad j = 1 \ldots M.$$

Note that the spheres are represented by listing their $M + 1$ positive radii, $\{\rho_j\}_{j=0}^M$, and $M$ center points, $\{q_i\}_{i=1}^M$. For ease of exposition we refer to $\mathbb{E}^n - \mathcal{W}$ as the zeroth obstacle, and center the workspace at the origin of our coordinate system.

The free space remains after removing all the obstacles from the workspace,

$$\mathcal{F} \triangleq \mathcal{W} - \bigcup_{j=1}^M \mathcal{O}_j.$$

For $\mathcal{F}$ to be a valid sphere world we must impose the additional constraint that all obstacle closures are contained in the interior of the workspace,

$$\rho_0 > 0 \quad \text{and} \quad \| q_i \| + \rho_i < \rho_0; \quad 1 \leq i \leq M,$$

and that none of them intersect,

$$\| q_i - q_j \| > \rho_i + \rho_j; \quad 1 \leq i, j \leq M.$$

**Theorem 4 ([10])** If the free space, $\mathcal{F}$, is a valid sphere world then there exists a positive integer $N$ such that for every $k \geq N$, for any finite number of obstacles, and for any destination point in the interior of $\mathcal{F}$,

$$\varphi = \sigma_d \circ \sigma \circ \varphi = \left( \frac{\gamma_k^d}{\gamma_k^d + \beta} \right)^{\frac{1}{d}}, \quad (4)$$

is a navigation function on $\mathcal{F}$.

Clearly, sphere worlds constitute a trivial task domain: there are more intuitive navigation schemes for such models whose proof of correctness would proceed more simply than ours. However, we next observe that the properties enjoyed by the navigation functions on sphere worlds remain invariant under diffeomorphism.

**Proposition 3.2 ([10])** Let $\varphi : \mathcal{M} \to [0, 1]$ be a navigation function on $\mathcal{M}$, and $h : f \to \mathcal{M}$ be an analytic diffeomorphism. Then

$$\tilde{\varphi} \triangleq \varphi \circ h,$$

is a navigation function on $f$.
Thus, our construction on a trivial "model space" automatically induces a correct solution for much more interesting robot navigation problems which are a "deformation" away from the model. Of course, there remains the difficult problem of constructing $h$. We conclude this section by reporting on the progress of our constructive deformation results.

A *star shaped set* is a compact connected subset of $\mathbb{E}^n$ for which there exists at least one "center point" from which every ray to its boundary intersects that boundary once; call such a boundary a *star shaped boundary*. A *star world* is a compact connected subset of $\mathbb{E}^n$ whose boundary is the disjoint union of a finite number of star shaped boundaries. Assume complete information about a star world in the form of an implicit function for each boundary component whose zero surface corresponds to that set, as well as certain other "tuning parameters" such as the smallest distance between two boundary components. In a recent report [13] we have constructed a family of analytic diffeomorphisms from any star shaped set into a suitable sphere world model. In a report presently in progress, we have extended this construction to topological sphere worlds whose boundary components are unions of star shaped boundaries.
4 Speculations on the Impact for Manufacturing

The body of work reviewed above has as its central aim the development of automatic procedures which link intention to action in the context of complex mechanical systems. Assuming a prescribed and uniform representation of the environment, this methodology promises to transform a rendering of some abstract goal in a specified syntax into a provably correct control algorithm. We may now review the practical implications of the requisite assumptions.

4.1 Representation

The natural control methodology presumes implicit representation of the environment. That is to say every object must be described as the level surface of some known scalar valued function. In contrast, it seems fair to say that most contemporary CAD/CAM systems rely upon splined surface representations to a much greater extent [3]. It is well known that moving back and forth between parametric and implicit representations is difficult [15], and in some sense a priori impracticable [6]. Thus any movement toward the natural control paradigm on the factory floor will require a much more concerted effort to develop reliable and computable implicit representations of solids with good combinatorial properties. We have begun to contemplate such an effort in our own work [13], but are convinced that meaningful advances will require a great deal of collaboration with experienced solids modeling researchers.

4.2 Syntax

Unfortunately, the appeal to analog models of computation in the solution of abstract problems has a strong element of the arcane. The expressive power of mathematical formulae seems impoverished relative to the straightforward and rich means of description afforded by ordinary language or even so drastically truncated versions as provided by the usual higher-level compilers to computer programmers. Yet, as has been described in the previous section, the advantages of the formalism include guarantees of correctness with respect to task specification as well as automatic synthesis of correct control laws. Moreover, the advent of real algebraic geometry based theorem provers [2] which similarly rely upon a formal mathematical syntax to encode statements about objects and more abstract relationships between them begins to suggest that this approach to robotics might link reasoning as well as specification to the control task. Finally, the freedom of expression afforded by an unconstrained linguistic user interface to an automation system belies the limited capabilities that all presently support: designers should be wary of false advertising. A formal interface (if it could be rendered "user friendly" without diluting the precision of expression) would afford users with constrained but predictable performance.
4.3 Computation

Established methodologies for determining and comparing the computational complexity of algorithms constructed for digital implementation have (as yet) no parallel within the analog realm: on the one hand the problem to be solved admits of radically different representations within the two domains; on the other hand, except in the case of linear systems where it is possible to speak of "time constants", there does not seem to be a canonical procedure for determining the "cost" in terms of time and resources of a general dynamical system. \(^1\) In part for these reasons, a precise comparison of the computational cost of the methodology embraced here with more traditional approaches to motion planning and control is difficult.

It is certain that this paradigm shifts a greater burden of computation away from the off-line planning system and toward the on-line real-time controller. For example, the computed torque algorithm — a control methodology based upon asymptotically exact tracking of off-line generated reference trajectories — entails a computation which grows linearly in the number of degrees of freedom [5]. In contrast, the evaluation of the gradient vector field required in algorithm (2) must entail a computational cost which grows exponentially in the degrees of freedom, since it has been shown that the general motion planning problem is \(\text{PSPACE} \) hard [14]. Yet it is by no means clear that this involves greater total computational complexity when the cost of the off-line reference trajectory generator has been added to the balance. We are presently investigating efficient parallel computation of the natural control algorithms presented in this paper [17].

\(^1\)Perhaps the current level of intense activity in the field of dynamical systems will give rise to such a theory: e.g., one might imagine a definition of the time complexity of a dynamical system involving its Lyapunov exponents; a possible candidate for characterizing the resource complexity might be the dimension of the state space (i.e. number of "integrators" required). It is not at all clear how these could be compared with the measures of combinatorial complexity developed for the analysis of algorithms in the digital domain.
References


