Template Based Control of Hexapedal Running

Uluç Saranli* and Daniel E. Koditschek*
[ulucs,kod]@eecs.umich.edu

Department of Electrical Engineering and Computer Science
The University of Michigan, Ann Arbor, MI 48109-2110, USA

Abstract

In this paper, we introduce a new hexapedal locomotion controller that simulation evidence suggests will be capable of driving our RHex robot at speeds exceeding five body lengths per second with reliable stability and rapid maneuverability. We use a low dimensional passively compliant biped as a "template" — a control target for the alternating tripod gait of the physical machine. We impose upon the physical machine an approximate inverse dynamics within-stride controller designed to force the true high dimensional system dynamics down onto the lower dimensional subspace corresponding to the template. Numerical simulations suggest the presence of asymptotically stable running gaits with large basins of attraction. Moreover, this controller improves substantially the maneuverability and dynamic range of RHex's running behaviors relative to the initial prototype open-loop algorithms.

1 Introduction

This paper concerns a new hexapedal running controller that promises to improve on the performance of prototype open-loop algorithms that presently drive our experimental hexapod robot, RHex [19]. Our emphasis on running is primarily motivated by the speed and efficiency afforded by dynamical modes of operation which are very difficult to achieve with more traditional, statically stable gaits for hexapedal robots [8, 11, 22]. Raibert's runners [15, 16] first demonstrated the advantages of such dynamical gaits in achieving performance beyond what is possible with purely kinematic algorithms. Later examples include the Scout class of quadrupeds [4, 5] and brachiating robots [14].

Over the last three decades, research in biomechanics [1, 7] has revealed that simple spring-mass models describe accurately the running motions of animals with very different sizes and morphologies [2, 3, 9]. Recently, a more formal model, the Spring-Loaded Inverted Pendulum(SLIP) has been introduced as a very useful tool in characterizing basic aspects of running, including stability and parameterizations of stable gaits [12, 21]. In this paper, we proceed one step further and adopt the SLIP model as a literal control target for running.

Toward this end, we decompose the control problem by introducing a bipedal extension to the basic SLIP model as a "template" — a simple dynamical system capturing the characteristic features of the task at hand [10]. In particular, the presence of two legs represents the alternating tripod gait which we adopt for our running controller. The lack of radial leg actuation in our experimental hexapod imposes a focus on explicit leg recirculation (rather than protraction) strategies leading to the introduction of a novel mechanism for its coordination. A natural correspondence between the passive radial compliance in RHex's legs and the SLIP model greatly reduces the active control effort required to achieve the target dynamics of the template.

Our resulting control architecture is an elaborated version of the template/anchor hierarchy of [20] with two levels. On "top" is a stride-to-stride level deadbeat controller for the discrete dynamical system obtained from the SLIP return map, affording a relatively simple task level interface for the command of mass center speed and height. Commands to the SLIP template impose carefully chosen parametric variations in a within-stride continuous time approximate inverse dynamics based hip torque controller that lies "beneath", attempting to force the dynamics of the robot to mimic the template as closely as possible. In the following sections, we provide systematic numerical evidence to suggest that the combination of these controllers will be capable of achieving reliably stable but highly maneuverable hexapedal running over a large range of speeds. An implementation on the physical

*Supported in part by DARPA/ONR Grant N00014-98-1-0747

RHex platform is in progress.

2 The Bipedal SLIP template

RHex's morphology introduces a number of fundamental constraints on feasible locomotion controllers. Most importantly, the limitation to one actuator per leg for a full 24 degree of freedom mechanism (6 dof in the body, 3 dof in each of the six legs — see [19]) imposes a severe degree of "underactuation", significantly exacerbated by the kinematic singularities around the standard operating configuration. As a result, controllers must rely on dynamic properties of the system, particularly the radial compliance in the legs, to achieve reasonable performance and range of behaviors. Our choice of template needs to capture these properties as well as the associated actuation limitations to ensure that its dynamics can be achieved with RHex's morphology.

In designing our controller, we primarily concentrate on the alternating tripod gait, which is adopted by the majority of hexapedal insects at high speeds [23]. It is characterized by the presence of two tripods, each formed by the front and back legs of one side and the middle leg of the opposite side. The tripods operate out of phase with each other, whereas the legs within a tripod are synchronized with each other. The resulting pattern describes a "virtual bipedal" gait, where the actions of the two tripods correspond to the two legs of a biped. For these reasons, a planar compliant biped forms the template for our locomotion controllers. In this section, we briefly review this template and its associated controllers. A much more complete treatment can be found in [18].

2.1 Hybrid System Model

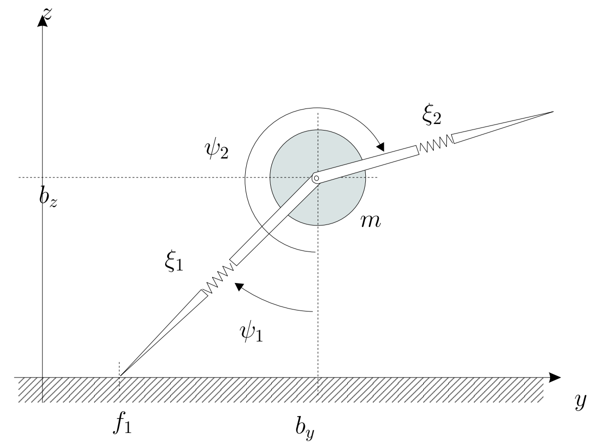

Figure 1: The Bipedal Spring-Loaded Inverted Pendulum(BSLIP) Model.

Figure 1 illustrates the planar Bipedal Spring-Loaded Inverted Pendulum (BSLIP) model. It consists of a point mass m, attached to two compliant massless legs that can freely rotate around the hip joint. Both legs incorporate passive springs as well as viscous damping. The mass is constrained to remain in the saggital plane, and is acted upon by gravity. Each leg has two alternating discrete modes — *stance* and *swing*.

Throughout the stance phase of a leg, its toe is fixed on the ground and the body is acted upon by the associated spring and damping forces. When the legs are in their swing phase, however, they do not affect the body dynamics. Their length and angle is governed by fully actuated first order dynamics, through which the touchdown angle and precompression can be controlled.

2.2 Control of Bipedal Gaits

Our bipedal locomotion controller has three major components: a finite state machine(FSM) to enforce leg alternation, a gait controller to regulate speed and height through proper choice of touchdown leg angles and precompressions as well as a recirculation controller to synchronize the stance and swing legs. For space considerations, this section only gives a brief overview of these components. Further details can be found in [18].

As a complement to the physical *modes* of a leg, discussed previously, the leg recirculation controller undertakes a succession of three states: *active*, *idle* or *recirculate*. The leg is active when it is in contact with the ground. It becomes idle when it lifts off and remains so until the touchdown of the other leg. Following this event, it starts recirculation in an attempt to achieve proper touchdown states designated by the gait regulation. A finite state machine governs transitions between these states.

In the spirit of Raibert's runners [15, 16], our controller regulates the speed and height of locomotion through a discrete set of command inputs modulating parameters of the system for each step. In our case, we use the touchdown angle, touchdown length (precompression) and liftoff length as our discrete command inputs. This choice of parameters is compatible with the radially passive nature of RHex's legs. Similar to our earlier work with a different set of inputs [20], we use a deadbeat strategy based on approximate plant inversion as our gait controller.

In consequence of the previously discussed morphological limitations of our experimental hexapod, RHex, leg motion during swing takes the form of recirculation rather than protraction. Consequently, the swing leg of our bipedal template also uses recirculation. Towards this end, we use a "mirror law" [6] to determine a target angle for the swing leg, which is then tracked by a local PD style feedback controller. The resulting feedback law is purely a function of the system state and ensures that the touchdown states designated by the deadbeat gait regulation are accurately achieved in a timely manner.

3 Hexapedal Running

3.1 The Spatial Hexapod Model

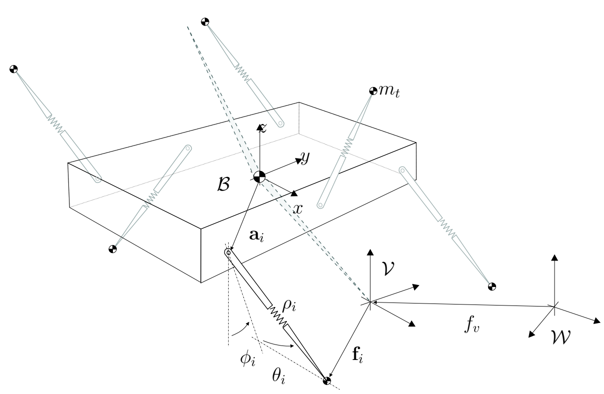

Figure 2: The compliant hexapod model.

Figure 2 illustrates the spatial compliant hexapod model that constitutes the basis for our controller design. Three reference frames are defined: $\mathcal{W}$ as the fixed inertial world frame, $\mathcal{V}$ as the virtual toe frame, located at the foot of a "virtual leg" and finally $\mathcal{B}$ as the body frame, affixed to the center of mass of the system. $\mathcal{V}$ and $\mathcal{W}$ have the same orientation except a yaw rotation around the z axis.

The model consists of a rigid body with six compliant legs whose attachment points are fixed at positions $\mathbf{a}_i$ in $\mathcal{B}$. The body has mass m and inertia matrix $\mathbf{I}_0$ with respect to $\mathcal{B}$. The orientation of the body is determined by the yaw γ, pitch α and the roll β angular degrees of freedom of the body.

Each leg has an associated mass $m_t \ll m$ to capture its flight dynamics. These masses introduce three additional degrees of freedom per leg: the radial extension ρ_i, the hip angle ϕ_i and the sideways leg angle θ_i. Each leg incorporates radial springs and viscous dampers. Similarly, the sideways leg degrees of freedom also have torsional springs and viscous dampers.

For legs in stance, the toe positions $\mathbf{f}_i$ are fixed on the ground and the rigid body is directly acted upon by the leg forces. In contrast, legs that are in flight do not exert forces on the body. Instead, the motion of the leg is governed by the dynamics of the associated toe mass under the influence of the leg forces. Moreover, the position and velocity of toe masses in flight become independent coordinates of the overall dynamics.

The morphology of this model accurately captures RHex's design. There are, however, two major differences in its dynamical properties. Firstly, the assumption that the toes remain stationary during stance is rather unrealistic. In fact, particularly at high speeds, leg slippage is one of the major limiting factors in RHex's performance. Secondly, simple linear leg compliance and damping models that we adopt are not experimentally verified and are likely to be inaccurate.

3.2 The Structure of the Controller

The design of our hexapedal running controller closely parallels the bipedal controller introduced in Section 2.2. An alternating tripod gait is imposed by associating each tripod with one of the biped legs and using the same finite state machine for leg alternation. Furthermore, the same gait level deadbeat controller is used to determine the desired touchdown commands at each step. There are, however, a few significant differences in the remaining components.

First of all, active radial actuation of the legs is not possible in RHex. As a consequence, it is not as simple to achieve the desired touchdown precompression. Toward this end, we introduce the idea of a "virtual toe" in Section 3.3, whose explicit placement in combination with appropriate modifications on the recirculation control yields the desired touchdown commands.

The remaining differences relate to the control of the stance tripod — also termed the active tripod. Section 3.4 briefly presents how we achieve the embedding of the BSLIP template through active control of the stance tripod. The embedding process also incorporates explicit control of the additional degrees of freedom of the hexapod such as the body orientation — yaw, pitch and roll — as well as lateral displacement of the body. In fact, the stability of these additional degrees of freedom turns out to be one of the most critical issues to impact the performance of our template based controller.

3.3 Virtual Toe Placement and Coordinates

At each step, our gait level BSLIP controller commands specific touchdown states to regulate the running speed and height. These commands must be realized by the underlying mechanism to yield convergence to the desired gait. Unfortunately, our limited actuation affordance over the hexapod does not admit precompression of its individual legs. It is hence unclear how the touchdown commands of the gait controller can be realized.

Our solution is to introduce the idea of a virtual toe, distinct from the physical toes of the hexapod. This also defines a virtual leg between the toe and the body center of mass, establishing a natural connection to individual legs of the biped.

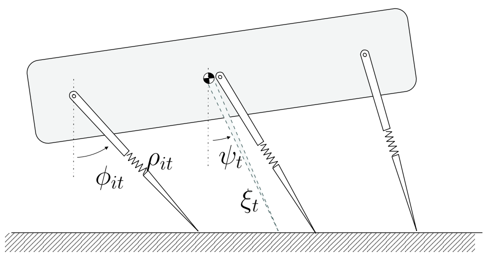

Figure 3: Touchdown kinematics of the recirculating tripod.

We use the recirculation of the swing legs in conjunction with explicit placement of the virtual toe to achieve the desired BSLIP touchdown states. Figure Figure 3 illustrates a side view of the hexapod together with the stance legs at touchdown. Given the commanded leg angle ψ_t and precompression ξ_t as well as the current body orientation, it is possible to solve the kinematics to compute target angles for the swing legs of the hexapod. Our recirculation controller for the hexapod takes the form of a mirror law, designed to achieve these target angles precisely at the moment of touchdown, while both avoiding premature transition into stance and satisfying the commands of the gait level controller[1].

The placement of the virtual foot also determines the new origin for the virtual toe frame $\mathcal{V}$. In order to facilitate the embedding of the BSLIP template, we define a new spherical coordinate system within $\mathcal{V}$: virtual toe coordinates. Figure 4 illustrates the associated definitions. Most importantly, there is an explicit correspondence between the saggital angle and virtual leg length coordinates and their counterparts

[1]See [18] for details of the derivations

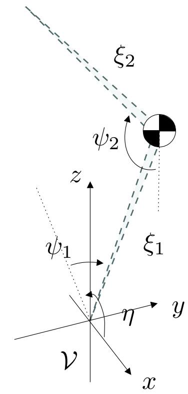

Figure 4: The virtual foot coordinates.

in Figure 1. In addition to this particular coordinatization of the translational degrees of freedom in the body, we use the yaw γ, pitch α and roll β to represent the orientation of the body.

3.4 Active Embedding of the Template

The goal of the embedding controller is to choose appropriate hip torque controls such that the dynamics of the hexapod center of mass mimic the passive stance dynamics of BSLIP as accurately as possible. The result is an effective reduction of the hexapod dynamics to the much simpler template dynamics, yielding the ability to regulate speed and height of locomotion using the gait level BSLIP controllers.

As a first step towards this end, it will be instrumental to derive the dynamics in virtual toe coordinates. Defining $\mathbf{c} := [\ \xi,\ \psi,\ \eta,\ \gamma,\ \alpha,\ \beta\ ]$, they take the form

$$\mathbf{M}(\mathbf{c})\ddot{\mathbf{c}} = f(\mathbf{c}, \dot{\mathbf{c}}) + \mathbf{K}\ , \tag{1}$$

where $f(\mathbf{c}, \dot{\mathbf{c}})$ are the unforced dynamics in polar coordinates. Furthermore, the forcing vector $\mathbf{K}$ is defined as

$$\mathbf{K} := (D_{\mathbf{c}}\boldsymbol{\phi})\boldsymbol{\tau} + (D_{\mathbf{c}}\boldsymbol{\rho})\mathbf{F}_r + (D_{\mathbf{c}}\boldsymbol{\theta})\boldsymbol{\tau}_\theta\ . \tag{2}$$

where $\boldsymbol{\tau}$, $\mathbf{F}_r$ and $\boldsymbol{\tau}_\theta$ are the hip actuation, radial spring force and sideways torque vectors for all the legs, respectively.

For exact embedding of BSLIP within the hexapod, a forcing vector of the form

$$\mathbf{K} = \left[U^*(\xi),\ 0,\ 0,\ M^*_\gamma,\ M^*_\alpha,\ M^*_\beta\right]$$

is required, where $U^*(\xi)$ denotes the potential law for the radial BSLIP spring. M^*_γ, M^*_α and M^*_β are desired effective torques on the Euler angle coordinates of the body orientation and are chosen through simple PD laws to stabilize the body to its neutral orientation.

The most obvious solution to (2) would be through inversion of $D_{\mathbf{c}}\boldsymbol{\phi}$. However, this turns out to be infeasible for a variety of reasons. First of all, the alternating tripod gait imposed by our controller admits at most three legs in contact with the ground at any time, yielding an underactuated system. Furthermore, our recirculation based strategy usually forces the system to go through configurations where all legs of the stance tripod are approximately parallel, decreasing control affordance. Finally, the hexapod almost always goes through midstance with vertical leg configurations and neutral body orientation, a singularity which is even more restrictive.

In order to address these problems, our strategy is to adopt a partial inversion of the dynamics. To this end, the structure of $D_{\mathbf{c}}\boldsymbol{\phi}$ suggests certain reductions (see [18] for a detailed discussion). In particular, we disregard the radial extension ξ as well as the sideways translation η from the inversion as the hip actuation offers very little if no affordance over these coordinates. Moreover, we share the remaining two degrees of actuation freedom between different subsets of the orientational degrees of freedom based on the properties of the jacobian.

Another complication arises from various limits on the torque commands at the hips. First of all, for practical applicability of our controller design, we impose a magnitude limit on the torque commands to match RHex's commercial actuator specifications. We also attempt to keep the stance legs on the ground as long as possible to avoid losing control affordance, imposing unilateral constraints on the allowable torque commands for each leg. The combination of these constraints yields the allowable torque space, defined as

$$\mathcal{T} := \{\ \boldsymbol{\tau}\ \mid\ \tau_{i,min} \leq \tau_i \leq \tau_{i,max}\ \}\ . \tag{3}$$

We now define two subsets of the virtual toe coordinates, $\mathbf{c}_1 := [\ \psi,\ \alpha,\ \beta\ ]^T$ and $\mathbf{c}_2 := [\ \psi,\ \gamma\ ]^T$. In inverting the associated submatrices of the overall jacobian, we prioritize the saggital plane angle ψ as the quality of the embedding is directly affected by the accuracy with which this component is satisfied. We hence require feasible torque solutions to lie in the set

$$\mathcal{T}_\psi := \{\ \boldsymbol{\tau}\ \mid\ (D_\psi\boldsymbol{\phi})\boldsymbol{\tau} + B_\psi = 0\ \}\ . \tag{4}$$

where B_ψ represents additional terms in (1).

Prior to computing the final solution, we first examine the determinant of $D_{\mathbf{c}_1}\boldsymbol{\phi}$. We use the submatrix corresponding to $\mathbf{c}_1$ whenever $\det(D_{\mathbf{c}_1}\boldsymbol{\phi}) > d_{min}$, and to $\mathbf{c}_2$ otherwise. In both cases, the final torque solution is computed by projecting the unconstrained solution from the jacobian inversion onto the allowable torque space of (3) along the feasible subspace defined in (4)[2].

$\mathbf{a}_1$	$\mathbf{a}_2$	$\mathbf{a}_3$
$[-0.1, -0.2, 0]$	$[-0.15, 0, 0]$	$[-0.1, 0.2, 0]$
$\mathbf{a}_4$	$\mathbf{a}_5$	$\mathbf{a}_6$
$[0.1, -0.2, 0]$	$[0.1, 0, 0]$	$[0.1, 0.2, 0]$

Table 1: Leg attachment points. Units are in meters.

m	m_t	I_x	I_y	I_z
$7kg$	$0.05kg$	$0.1kgm^2$	$0.029kgm^2$	$0.11kgm^2$

Table 2: Dynamical parameters of the hexapod

4 Simulation Studies

In this section, we use numerical simulations to show that our template based controller achieves asymptotically stable locomotion for a wide range of forward speeds. We also characterize in simulation, stability properties of the associated limit cycles.

4.1 Numerical Environment

The software package that we use for the forward integration of the system described in Section 3.1 is *SimSect*, a generic hybrid dynamical system simulation environment [17]. The results of the following sections were obtained using kinematic and dynamic parameters that match RHex's morphology as closely as possible. Despite differences in the surface contact model as well as the lack of experimental validation of our leg compliance and damping models, we believe that the simulation results we present will be qualitatively applicable to physical implementations.

Table 1 specifies the attachment points in $\mathcal{B}$ for the legs of the hexapod, sufficient to determine the morphology of the hexapod. Similarly, Table 2 details the dynamical parameters of the rigid body. Finally, the leg model parameters in Table 3 complete the specification of the dynamical model.

4.2 Nature of Stable Orbits

In contrast to our earlier control strategies which were exclusively based on time dependent reference trajectories [19], the action of the template based controller

[2]See [18] for details on how this projection is performed.

ρ_0	$k_{\rho a}$	$k_{\theta a}$	$d_{\rho a}$	$d_{\theta a}$
0.17 m	1500 N/m	300 Nm/rad	12 Ns/m	1.2 Nms/rad
θ_0	$k_{\rho b}$	$k_{\theta b}$	$d_{\rho b}$	$d_{\theta b}$
0 rad	2000 N/m	400 Nm/rad	16 Ns/m	1.6 Nms/rad

Table 3: Leg model parameters. Spring constants k and damping coefficients d with the subscript b denote those of the middle legs, whereas parameters with the subscript a correspond to the front and back legs.

on the spatial hexapod model results in a completely autonomous dynamical system devoid of all time dependency. We have been able to identify through simulation, asymptotically stable limit cycles of this system which seem to be unique for each different gait level goal setting. Furthermore, these limit cycles all seem to have the same structure and characteristic features.

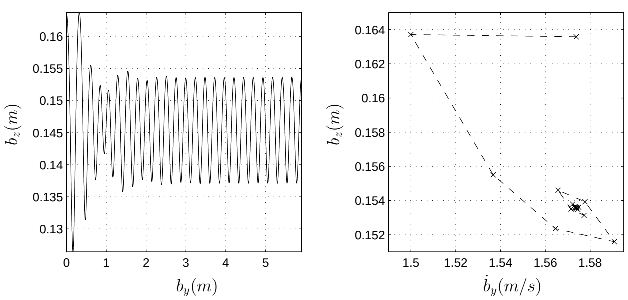

Figure 5: Illustration of the attracting limit cycle for an example run for $\dot{b}_y^* = 1.6m/s$. The left figure shows the saggital plane position of the robot while the right figure shows the progression of the gait level apex state towards the fixed point in the $\dot{b}_y - b_z$ plane.

First of all, despite the mirrored morphology of the left and right tripods, the projection of the limit cycle onto the saggital plane coordinates (forward velocity and body height) exhibits period one behavior from one step to the next. Figure 5 illustrates this aspect of an example run.

On the other hand, projection onto the roll and yaw degrees of freedom reveals period two behavior as a consequence of the alternation between the left and right tripods. Fortunately, this does not seem to affect the task level stability in the saggital plane coordinates, which were always observed to be period one. All the limit cycles we have obtained using the template based controller exhibit these properties.

4.3 Stability and Basins of Attraction

In this section, we present simulation evidence to show that the stable limit cycle described above can be effectively adjusted by changing gait level commands to the SLIP template, resulting in an effective control of forward velocity. Specifically, we summarize the results of careful numerical study indicating that the basins of attraction associated with these cycles are sufficiently large to admit smooth control of forward velocity of locomotion.

In identifying stability properties of a particular goal speed, we would ultimately need to investigate the entire 12 dimensional space of rigid body initial conditions. Fortunately, it is possible to reduce the size of this space by considering certain symmetries in the system. In particular, we do not need to consider the horizontal translation and yaw initial conditions of the robot. Furthermore, since gait characteristics are well captured by the discrete return map, we drop another dimension (choosing to work with "apex" coordinates that specifically eliminate the vertical velocity component). As a consequence, the dimension of the space of initial conditions is reduced to eight. However, even with this reduced space, it is very costly to characterize carefully the basins of attraction due to the computational cost of the required simulations. For purposes of presentation we project onto two different pairwise combinations of these eight dimensions, an approximation to the basin around four different speed settings, illustrated in Figures 6 and 7.

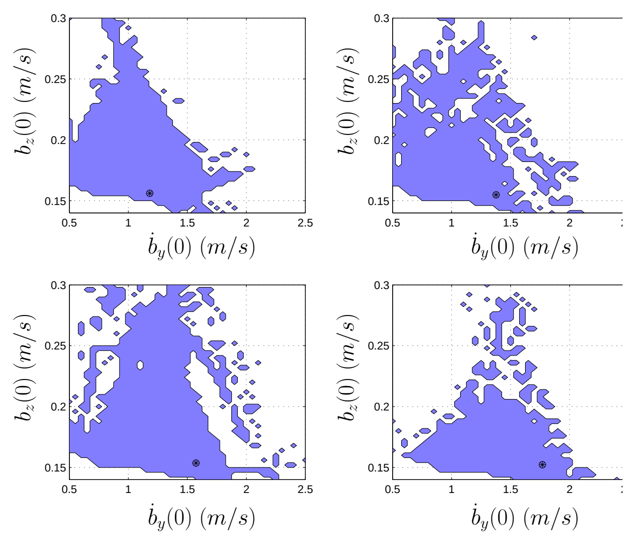

Figure 6: Projection of the basin of attraction for 4 different speed goals $\dot{b}_y^* \in \{1.2, 1.4, 1.6, 1.8\}$m/s onto BSLIP apex coordinates. For each goal setting, the filled circle indicates the stable limit cycle.

In particular, Figure 6 concerns the saggital plane velocity and height, which also correspond to the task level coordinates of the BSLIP template. The surprisingly large basins of attraction associated with four different speed settings suggest that smooth control of forward velocity is possible.

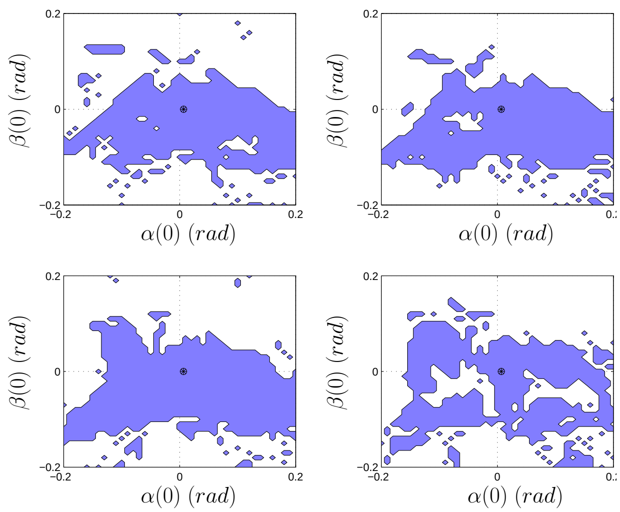

Figure 7: Projection of the basin of attraction for 4 different speed goals $\dot{b}_y^* \in \{1.2, 1.4, 1.6, 1.8\}$m/s onto pitch and roll coordinates. For each goal setting, the filled circle indicates the stable fixed point.

Similarly, Figure 6 illustrates the projection of the basin of attraction onto two most critical orientational degrees of freedom of the rigid body. Even though roll instability is the predominant source of failure for our controller, the basins of attraction are reasonably large in both the pitch and roll directions. This relatively strong stability suggests that practical implementations on RHex may be feasible.

5 Conclusion

In this paper, we develop a novel model based locomotion controller that is capable of achieving asymptotically stable hexapedal running for a large range of speeds. We demonstrate the efficacy of this controller by its application to a hybrid Lagrangian model of the hexapedal robot, RHex [19]. A bipedal extension of the well-studied Spring-Loaded Inverted Pendulum model is used as the dynamical motion template and command interface for hexapedal locomotion. This is complemented by an inverse dynamics style controller designed to embed the template dynamics within the hexapod model, effectively reducing the model dynamics to those of the template. Simulation studies yield convincing evidence that the combination of a gait level controller acting on the template, with our model based embedding strategy is sufficient to achieve asymptotically stable hexapedal running.

The natural next step is implementation on RHex. However, there are a number of challenges in realizing such an implementation. First of all, the simple fixed toe model adopted by our embedding controller is rather unrealistic and more accurate extensions need to be incorporated to ensure practical applicability. Similarly, passive properties of RHex's legs need to be accurately modeled prior to an actual implementation. Some recent results promise to address some of these issues [13].

Further difficulties arise from the high bandwidth state feedback required by our controller. In fact, the resulting controlled dynamics are fully autonomous and have no explicit time dependence. Clearly, any implementation of such an algorithm requires accurate and reliable sensing that will be feasible only after significant effort, now in progress, devoted to the design and implementation of careful state estimators over RHex's expanding sensor suite. A careful characterization of our controller's performance under noise as well as the implementation of correspondingly accurate sensor hardware and software for RHex needs to be completed prior to an experimental implementation.

In summary, there is still a long research path to our ultimate goal of building a fully autonomous legged platform capable of surviving a large range of outdoor environments for extended periods of time. However, we believe that our work represents an important step in this direction and begins to develop some of the tools and concepts that are necessary to achieve this goal.

Acknowledgments

We thank Richard Altendorfer for his insight in understanding the structure of the Jacobians. This work was supported in part by DARPA/ONR Grant N00014-98-1-0747.

References

[1] R. M. Alexander and A. S. Jayes. Vertical movement in walking and running. *Journal of Zoology, London*, 185:27–40, 1978.

[2] R. Blickhan. The spring-mass model for running and hopping. *Journal of Biomechanics*, 22:1217–1227, 1989.

[3] R. Blickhan and R. J. Full. Similarity in multilegged locomotion: Bouncing like a monopode. *Journal of Comparative Physiology*, A. 173:509–517, 1993.

[4] M. Buehler, R. Battaglia, A. Cocosco, G. Hawker, J. Sarkis, and K. Yamazaki. SCOUT: A simple quadraped that walks, climbs and runs. In *Proceedings of the IEEE International Conference On Robotics and Automation*, pages 1707–1712, Leuven, Belgium, May 1998.

[5] M. Buehler, A. Cocosco, K. Yamazaki, and R. Battaglia. Stable open loop walking in quadruped robots with stick legs. In *Proceedings of the IEEE International Conference On Robotics and Automation*, pages 2348–2353, Detroit, Michigan, May 1999.

[6] M. Buehler, D. E. Kodischek, and P. J. Kindlmann. Planning and Control of Robotic Juggling and Catching Tasks. *International Journal of Robotics Research*, 13(2):101–118, April 1994.

[7] G. A. Cavagna, N. C. Heglund, and C. R. Taylor. Mechanical work in terrestrial locomotion: Two basic mechanisms for minimizing energy expenditure. *American Journal of Physiology*, 233(5):R243–R261, 1977.

[8] K. S. Espenschied, R. D. Quinn, H. J. Chiel, and R. D. Beer. Biologically-Inspired Hexapod Robot Control. In *Proceedings of the 5th International Symposium on Robotics and Manufacturing: Research, Education and Applications (ISRAM' 94)*, Maui, Hawaii, August 1994.

[9] C. T. Farley, J. Glasheen, and T. A. McMahon. Running springs: Speed and animal size. *Journal of Biomechanics*, 185:71–86, 1993.

[10] R. J. Full and D. E. Koditschek. Templates and Anchors: Neuromechanical Hypotheses of Legged Locomotion on Land. *Journal of Experimental Biology*, 202:3325–3332, 1999.

[11] S. Hirose. A study of design and control of a quadruped walking machine. *International Journal of Robotics Research*, 3(2):113–133, 1984.

[12] T. A. McMahon and G. C. Cheng. The mechanics of running: How does stiffness couple with speed. *Journal of Biomechanics*, 23(Suppl. 1):65–78, 1990.

[13] E. Z. Moore. Leg design and stair climbing control for the rhex robotic hexapod. Master's thesis, Dept. of Mechanical Engineering, McGill University, November 2001.

[14] J. Nakanishi, T. Fukuda, and D. E. Koditschek. A brachiating robot controller. *IEEE Transactions on Robotics and Automation*, 16(2):109–123, 2000.

[15] M. H. Raibert. *Legged robots that balance*. MIT Press, Cambridge MA, 1986.

[16] M. H. Raibert and J. K. Hodgins. Animation of dynamic legged locomotion. *Computer Graphics*, 25(4):349–58, July 1991.

[17] U. Saranli. SimSect Hybrid Dynamical Simulation Environment. Technical Report CSE-TR-436-00, UM, Ann Arbor, MI, 2000.

[18] U. Saranli. *Dynamic Locomotion with a Hexapod Robot*. PhD thesis, Computer Science and Engineering, University of Michigan, 2002.

[19] U. Saranli, M. Buehler, and D. E. Koditschek. RHex: A Simple and Highly Mobile Robot. *International Journal of Robotics Research*, 20(7):616–631, July 2001.

[20] U. Saranli, W. J. Schwind, and D. E. Koditschek. Toward the Control of a Multi-Jointed, Monoped Runner. In *Proceedings of the IEEE International Conference On Robotics and Automation*, volume 3, pages 2676–82, Leuven, Belgium, May 1998.

[21] W. J. Schwind. *Spring Loaded Iverted Pendulum Running: A Plant Model*. PhD thesis, Electrical Enginnering: Systems. The University of Michigan, 1998.

[22] S. M. Song and K. J. Waldron. *Machines that walk: The Adaptive Suspension Vehicle*. MIT Press, Cambridge MA, 1989.

[23] L. H. Ting, R. Blickhan, and R. J. Full. Dynamics and static stability in hexapedal runners. *The journal of Experimental Biology*, 197:251–269, 1994.