Kinematic Leg Design in an Electromechanical Robot

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Abstract— **An unconventional kinematic leg design is synthesized from the reconciliation of three dynamic task specifications, represented as constrained optimization problems derived from energetic first principles. Numerical optimization and judicious decoupling of design parameters suggest that monotonically decreasing effective mechanical advantage with leg extension can yield substantial performance benefits relative to conventional practice.**

I. INTRODUCTION

For a robot doing work in the physical world, the dynamical interactions between the machine and its surroundings (humans, objects, and the ground) govern the system's performance capabilities. This was first apparent in the pursuit of manipulator force control [1]–[5] and refined in actuators designed not only to exert forces, but to have forces exerted upon them [6]–[8]. In legged locomotion, there are obvious advantages to having an actuator allowing for transparent [9] and flexible (subserving in part or whole a variety of functions, for instance, "motors, brakes, springs, and struts" [10]) manipulation of the system's natural dynamics.

The design of a machine capable of harnessing its natural dynamics in pursuit of its tasks involves the simultaneous representation and selection of components across a diversity of physical modalities spanning compliance properties, power characteristics, materials, and kinematics—first to construct the mechanical stage upon which the dynamics can be played out, and then to direct effectively their recruitment for the task at hand. Notwithstanding their intimate coupling in the physical platform, parameters representative of these distinct physical modalities are largely optimized individually or pairwise at a given operating point. Leg design is considered in [11]–[14], compliance in [15]–[18] and actuator selection in [7], [19]–[21].

In the spirit of this workshop, the main contribution of this paper is an effort to explore the formal representation and rational solution of a simple prototypical problem in the design of a dynamical robot from first energetic principles. A 1 DOF template [22] will be analyzed during a single stance event. This simple model exposes many of the core design challenges centered around the transfer and management of energy by considering a particular [23] (but nearly ubiquitous in dynamical locomotion [24], [25]) hopping behavior. Formalizing in this interlocking manner the task specifications suggests a more effective but unconventional leg design that allows a fixed power source to deliver twice as much kinetic energy to the body, an order of magnitude decrease in losses due to modeled collisions, and double the energy storage of the spring, relative to the conventional alternative.

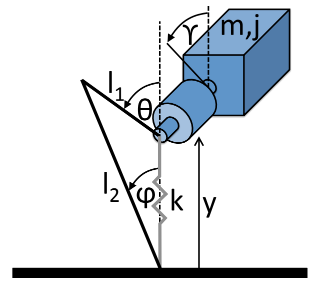

Fig. 1. Model for the Single DOF Hopping Robot

II. DESIGN OBJECTIVES

In the pursuit of highly dynamic machines that can manipulate themselves and their environment in useful ways, management of the system's energy is of utmost importance [26]. During stance, kinetic energy must be transduced by the motor from stored chemical potential energy into kinetic energy in the body. This kinetic energy should then be retained by minimizing losses in the system (especially due to collisions). Moreover, the energy should be harvested from stride to stride to improve efficiency and peak energy. This management of energy presents three design objectives:

1) effective conversion to mechanical energy
2) mitigation of collision losses
3) harvest of energy from stride to stride

These thematically distinct but parametrically intertwined design objectives will now be evaluated in the context of the template described in Section III, then presented in Section IV as three distinct task specifications that can be individually optimized and then rationally reconciled.

III. MODEL

The system under consideration is a single degree of freedom vertical hopping robot of a kind originally proposed in [24] and formally studied in [23]. As in [27], the model exposes the electromagnetic actuator dynamics, but now only to a kinematically simplified 1 DOF version of the hopping mechanics, in order to focus as narrowly as possible on the role of the infinitesimal kinematics in task performance.

The body consists of a point mass, m, motor with inertia, j, and a massless leg with two rigid links of length l_1 and l_2. The motor's angle, γ is related to the angle of the first link, θ according to the motor's gear ratio:

$$G_{motor} = \frac{\gamma}{\theta} \tag{1}$$

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The angle of the second link with respect to vertical is ϕ. The robot is constrained to operate on a fixed vertical axis, so the only state variable is the robot's height, y, which will be related to the various link angles in the following subsection.

This analysis will assume that actuator selection is already performed according to the general principles of [7], [9]. Specifically, torque density ($\frac{Nm}{kg}$) is maximized according to [19] to improve transparency.[1]

The main benefits of series elastic actuation [8] are:

1) decreased reflected motor inertia
2) stable force control
3) elastic energy storage

The assumed actuator selection mitigates the first problem (detailed in Section IV). Stable force control is highly desirable, but typically comes at the cost of dynamically isolating the motor, decreasing both the sensing and actuation bandwidth. New designs allow this isolation to be modulated using variable compliance [16], but other tradeoffs must be made[2]. Regarding energy storage, this paper focuses on the benefits of adding an elastic element taking the form of a linear spring with stiffness, k, attached in parallel with the motor. This allows the motor to do work on the spring when the toe is not touching the ground, and enables the harvest of energy from stride to stride (discussed further in Section III). This is a much more effective means of energy storage since the motor is not relied upon to provide the necessary reaction forces to maintain compression (or tension) in the spring. There is therefore no obvious advantage to including a series spring, and it will be omitted in this treatment to focus on the other more complementary modalities.

A. Leg Kinematics

A slider-crank (RRRP) linkage is used to represent the hip transmission (the mechanical connection between the revolute joint angle and prismatic leg shaft) since it is one of the simplest closed linkages [29] and includes enough design freedom to place the (topologically unavoidable) pair of kinematic singularities as desired within the jointspace. We depict the salient features of this notional transmission element in Fig. 3 and list the specific physical parameters in Table 1, taken from a machine currently under development, in order to afford near-term empirical validation of the design ideas presented in this paper, shown in Fig. 7.

From this generic selection of kinematic parameters arises an important distinction between our study and the convention in recent locomotion literature, as the driven link is able to travel according to $\theta \in [0, \pi]$ instead of the typical $\theta \in [\frac{\pi}{2}, \pi]$ [11]–[13], [30]. With a notional transmission element in place, the key parameter variation we now study

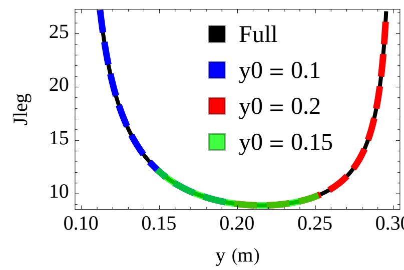

Fig. 2. J_{leg} for $y \in [y_0, y_0 + y_{travel}]$

is the run-time decision of where to place the initial leg extension at touchdown, y_0, with a choice of gearing and spring constant (to be optimized subsequently in the tasks below) that will allow some appropriate travel of length $y_{travel} = 0.1$. While the interval is fixed, this freedom of choice in y_0 still exposes a large variation of mechanical advantage in consequence of its correspondingly diverse placements relative to the endpoint singularities. In each case, G_{motor} is varied, effectively stretching or shrinking the magnitude of J_{total} to find the optimal solution for a trial. One intermediate and two extreme examples of the portion of J_{leg} exposed by choice of y_0 are depicted in Fig. 2. Observe that the case where $y_0 = 0.2$ represents conventional leg kinematics as it is the only operating point that exists entirely inside $\theta \in [\frac{\pi}{2}, \pi]$.

We derive the inverse kinematics by using leg position constraints in the horizontal and vertical directions

$$l_2 \sin\phi = l_1 \sin\theta \tag{2}$$

$$l_2 \cos\phi - y = l_1 \cos\theta, \tag{3}$$

so that θ can be expressed as a function of y:

$$f(y) := \arctan\left(\frac{-(l_1^2 - l_2^2 + y^2)}{\sqrt{-l_1^4 - (l_2^2 - y^2)^2 + 2l_1^2(l_2^2 + y^2)}}\right). \tag{4}$$

The resulting inverse infinitesimal kinematics

$$J_{leg}(y) := \frac{df}{dy} = \frac{-l_1^2 + l_2^2 + y^2}{y\sqrt{-l1^2 - (l_2^2 - y^2)^2 + 2l_1^2(l_2^2 + y^2)}} \tag{5}$$

can now be composed with the gear ratio, to express the complete map from motor shaft output torque to vertical toe force

$$J_{total}(y) = G_{motor} J_{leg}(y) \tag{6}$$

Accounting for this composition constitutes an obvious but frequently neglected modeling step, as the the motor gear ratio (G_{motor}) and leg Jacobian (J_{leg}) are typically considered to reside in two different domains of design, [21], [31].

B. Equations of Motion

The system's kinetic energy can be expressed as

$$T = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}j(J_{total}(y)\dot{y})^2 \tag{7}$$

and potential energy

$$V = mgy + \frac{1}{2}k(y_0 - y)^2 \tag{8}$$

where y_0 is the resting length of the spring. The Lagrangian can then be calculated according to $L = T - V$. The only external forces are due to the motor. We now assume that

[1] This improved transparency is accomplished by minimizing or even eliminating the motor's gearbox for given torque requirements and surrendering a significant fraction of the robot's mass to the actuator, improving K_m [7], the thermal capacitance, and the thermal conductance. This increased thermal budget can prolong the tipping point when the benefits of a vascular system such as [28] outweigh its complexity, but such considerations are outside the scope of this paper.

[2] The complexity of adding a means of varying the compliance must be weighed against its benefits. Additionally, since compliance cannot be varied arbitrarily quickly, the operating regime must be chosen carefully, but these considerations are outside the scope of this paper.

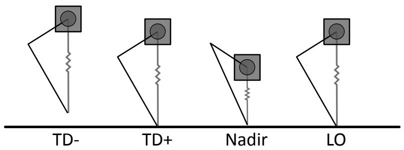

Fig. 3. Stance Events

the motor will be operated at full capacity — at maximal permissible (i.e. consistent with currents that respect thermal limits) constant terminal voltage, v, which implies that the motor shaft output follows a typical speed-torque curve [31]:

$$\tau = 1 - \frac{\tau_{max}}{\dot{\gamma}_{nl}}\dot{\gamma}. \tag{9}$$

where $\dot{\gamma}_{nl} = K_v v$ and $\tau_{max} = \frac{i}{K_v}$. K_v is the motor speed constant, and v and i are the supply voltage and current respectively (which we assume are algebraically related in consequence of vanishingly faster electrical time constants). The external force on the body exerted by the motor is

$$F_{ext} = J_{total}(y) \cdot (\tau_{max} - \frac{J_{total}(y)\tau_{max}}{\dot{\gamma}_{nl}}\dot{y}) \tag{10}$$

and the equations of motion in stance can be written out by expanding the Euler-Lagrange operator,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = F_{ext} \tag{11}$$

IV. DYNAMIC TASK SPECIFICATIONS

The three design objectives introduced in Section II will now be considered with the model to formalize the three task specifications as distinct constrained optimization problems.

A. Effective Conversion to Mechanical Energy

Fig. 2 depicts the distinctly different J_{leg} profiles achievable by choice of operating point, y_0, yielding an effective mechanical advantage that is, qualitatively speaking, either monotonically decreasing, unimodal, or monotonically increasing as the leg travel index (y_0) is varied. The central object of study in this paper is the consequent modulation of the ground reaction force felt at the motor over the course of this vertical travel, with the goal of allowing it to operate in a higher power regime, resulting in greater work performed. However, the closed loop dynamics (Eqn. 11) is a highly nonlinear dissipative second order system for which no closed form solutions can be expected, hence we resort to numerical analysis in this paper. The integral corresponding to the motor's output energy is obtained by integrating the external force due to the motor (Eqn. 10) from nadir to liftoff:

$$E_N^{LO} = \int_{y_0}^{y_0+y_{travel}} J_{total}(y)(\tau_{max} - \tfrac{J_{total}(y)\tau_{max}\dot{y}}{\dot{\gamma}_{nl}})dy \tag{12}$$

The first dynamic task specification can now be formalized as the optimization of Eqn. 12 with respect to the operating point, y_0, and gearing, G_{motor}. The spring constant, k, is monotonic with liftoff energy since it represents initial strain energy. It is therefore fixed at $k = 0$ (worst case scenario) since this represents the circumstances when getting kinetic energy from the motor is most critical.

For the physical parameters listed in Table 1, numerical optimization results in optimal $E_N^{LO} = 2.89J$ at $y_{01}^* = 0.104$ and $G_{motor}^* = 1.75$ (shown in Fig. 4). We will now fix the gearing at G_{motor}^* because the final two objectives turn out to be insensitive to it, as discussed further below.

B. Mitigation of Collision Losses

While this system is only considered during a single stance event, the inclusion of a TD- state means that collisions (due to instantaneous changes in motor velocity) can be modeled. The system is assumed to collide plastically with the ground at touchdown, and the energy lost to this collision can be calculated with a simple momentum balance, very similar to [15]. T_{loss}, a function of the pre-collision energy, T^-, is:

$$T_{loss} = (1 - \alpha)T^- \tag{13}$$

where α is the collision efficiency:

$$\alpha := 1 - \frac{j J_{total}^2|_{y=y_0+y_{travel}}}{m + j J_{total}^2|_{y=y_0+y_{travel}}} \tag{14}$$

This quantity can be optimized analytically using the expression for J_{total} derived in Eqn. 6. Observe that the spring constant, k, does not appear in Eqn. 14 because the collision it represents is being modeled as an impulse. Observe, as well, that α is degenerate in the sense that $\frac{\partial \alpha}{\partial G motor} > 0$ so that the extremum with respect to y_0 is G_{motor}-invariant.[3] The result of the numerical study is shown in Fig. 5 is a new optimal $y_{02}^* = 0.115$ ($\alpha = 0.976$).

C. Energy Harvest from Stride to Stride

After touchdown, the parallel spring can be used to harvest the remaining kinetic energy from flight and store it temporarily in strain. Additionally, the motor can be used to do work on the spring from TD+ to nadir. The third task then seeks to maximize the spring's strain energy from TD+ to nadir:[4]

$$E_{TD}^N = \int_{TD+}^{Nadir} J_{total}(y)(\tau_{max} - \tfrac{J_{total}(y)\tau_{max}\dot{y}}{\dot{\gamma}_{nl}})dy \tag{15}$$

which is actually evaluated from $y_0 + y_{travel}$ to y_0, since solutions that do not use the whole interval can be shown to be suboptimal. A further condition is imposed such that the motor is always able to overpower the spring:

$$J_{total}(y) \cdot \tau_{max} > k(y - y_0), \forall y \in (y_0, y_0 + y_{travel}) \tag{16}$$

For the physical parameters listed in Table 1, and $G_{motor} = G_{motor}^* = 1.75$,[5] numerical optimization of Eqn.

[3]This can be seen directly by observing that α is monotone in J_{total}^2, which, in turn is monotone in G_{motor}^2. In consequence, the solution to $\frac{\partial \alpha}{\partial y_0} = 0$ occurs along the entire "ridge" (y_0^*, G_{motor}) where $y_{02}^* := 0.115$. We show a cross section of this "ridge" in Fig. 5 for optimal value of $G_{motor}^* := 1.75$ obtained in the previous optimization.

[4]The kinetic energy remaining from flight is ignored, but if the spring is selected for maximal energy storage, some harvested energy can be traded for battery energy from the motor.

[5]E_{TD}^N is monotone with G_{motor}, so the former must be fixed, logically at G_{motor}^*

Table 1: Physical Model Parameters		
Parameter	Symbol	Value
Mass	m	$1kg$
Motor Inertia	j	$10^{-4}kgm^2$
Motor Stall Torque	τ_{max}	$3.15Nm$
Motor No Load Speed	$\dot{\gamma}_{noload}$	$83.6\frac{rad}{s}$
Link 1 Length	l_1	$0.1m$
Link 2 Length	l_2	$0.2m$

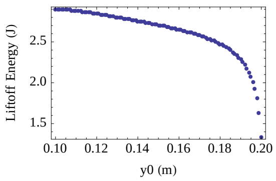

Fig. 4. Liftoff energy at y_0

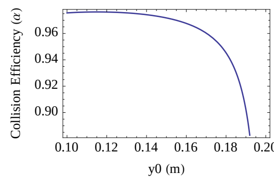

Fig. 5. Collision efficiency (α) at y_0

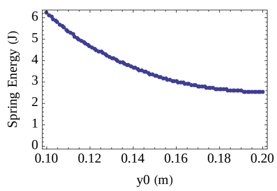

Fig. 6. Spring energy at y_0

15 while ensuring Eqn. 16 results in maximal $E_{TD}^N = 6.23J$ at $y_{03}^* = 0.1$ and $k^* = 1245N/m$, shown in Fig. 6.

V. CONCLUSION

A summary of the results from optimizing the individual tasks is presented in Table 2. While the three parameters of interest initially appear to be overconstrained by their participation in these potentially conflicting multiple objectives, further analysis suggests that G_{motor} and k can be reasonably decoupled (optimized once) while the intrinsically coupled effects of the kinematics, J_{leg} (indexed by y_0) turn out to play a synergistic role across all three objectives. In sum, the analysis suggests broadly more favorable performance when the effective mechanical advantage is monotonically decreasing with y_0. This unconventional design is soon to be evaluated using the apparatus of Fig. 7.

Table 2: Summary of task specification results			
	y_0^*	G_{motor}^*	k^*
Task 1	0.104	1.75	Use k=0
Task 2	0.115	Invariant	Invariant
Task 3	0.1	Use G_{motor}^* from Task 1	1245 N/m

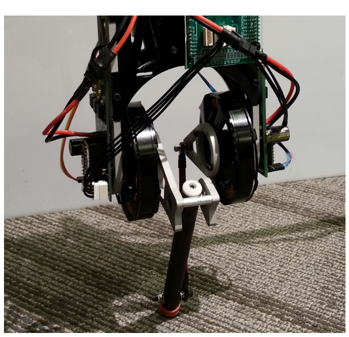
Fig. 7. Prototype of physical machine, cropped to highlight leg design

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